The Mann-Whitney U-Test

Sometimes distributions of variables do not show a normal distribution, or the samples taken are so small that one cannot tell if they are part of a normal distribution or not. Using the t-test to tell if there is a significant difference between samples is not appropriate here.
The Mann-Whitney U-test can be used in these situations. This test can be used for very small samples (between 5 and 20). It can also be used when the variable being recorded is measured using an arbitrary scale which cannot be measured accurately (e.g. a colour scale measured by eye or a behavioural trait such as aggression).

The following example will illustrate the method.

The size of leaves taken from bramble bushes were measured to see if there is a difference between the size of the leaves growing in full sunlight and those growing in the shade.

	Width of leaf / cm							
Sunlight	6.0	4.8	5.1	5.5	4.1	5.3	4.5	5.1
Shade	6.5	5.5	6.3	7.2	6.8	5.5	5.9	5.5

The Mann-Whitney U-test is chosen because the sample size is so small it is not clear if these are samples taken from normally distributed data.

1. Set up the Null Hypothesis: There is no difference between the leaves taken from the sunlit bramble and the shaded bramble.
   Alternative Hypothesis: There is a difference between the leaves taken from the sunlit bramble and the shade bramble.

2. Let $\mathbf{n_1}$ be the size of the smallest sample and $\mathbf{n_2}$ be the size of the biggest sample.
   In this example both are of the same size so it does not matter which you choose.

$$\mathbf{n_1} = 8 \text{ and } \mathbf{n_2} = 8$$

3. Rank all the values for both samples from the smallest (=1) to the largest. Set them up as shown in the table below.

Sunlight	Rank	Rank	Shade
4.1	1		
4.5	2		
4.8	3		
5.1	4.5		
5.1	4.5		
5.3	6		
5.5	8.5		
		8.5	5.5
		8.5	5.5
		8.5	5.5
		11	5.9
6.0	12		
		13	6.3
		14	6.5
		15	6.8
		16	7.2
R_1 =	**41.5**	**94.5**	**= R_2**

Note where the values are the same and share the same rank, take an average of the rank values.

4. Total the ranks of each sample $\mathbf{R_1}$ and $\mathbf{R_2}$ (see the bottom of the table above).

5. Calculate the **U** values for both samples:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 = (8x8) + \frac{8x9}{2} - 41.5 = 58.5$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 = (8x8) + \frac{8x9}{2} - 94.5 = 5.5$$

6. Use the table to find the critical value for the U statistic at the 5% level for samples of this size ($\mathbf{n_1}$ = 8 and $\mathbf{n_2}$ = 8).

   $\mathbf{U_{crit}}$ = 13

7. Reject the Null Hypothesis if the smallest value of $\mathbf{U_1}$ or $\mathbf{U_2}$ is below $\mathbf{U_{crit}}$. In this case $\mathbf{U_2}$ is below 13 we can reject the Null Hypothesis and accept the Alternative Hypothesis. The difference between the size of the bramble leaves in the light and the dark is significant for P>0.05. Bramble leaves in the dark seem to be significantly bigger.